blue-green to blue by nitric and sulphuric acids, and generally blue-green with iodine in iodide of potassium (in the solid state).

On isolation of the yellow constituent of enterochlorophyll by saponification and extraction with petroleum ether, I found that it generally showed only one band, or sometimes two, but these bands generally gave different measurements from those of plant chlorophyll.

To see whether symbiotic algæ were present in the organs yielding enterochlorophyll, I examined fresh frozen sections, or portions of the organ teased out in salt solution, but the results were negative. On steeping such preparations, first in alcohol, then in weak solution of caustic soda, and neutralising with acetic acid, and afterwards testing with a solution of iodine in iodide of potassium and with Schultze's fluid, I never obtained evidence of the presence of starch or cellulose. Hence, apart from the absence of symbiotic algæ under the microscope, this result negatives their presence and also that of food chlorophyll. The morphology of enterochlorophyll was studied in similar preparations, and on the whole it appears to be present dissolved in oil globules and in granules, both of them enclosed in the epithelium lining the liver tubes. It also occurs dissolved in the protoplasm of the liver cells, and these appearances vary slightly in different cases.

It would therefore appear that enterochlorophyll is built up in the organ containing it; that it is a chlorophyll, of which there are several in animals, and that it is composed of two constituents, of which one resembles closely the corresponding constituent of plant chlorophyll, while the other is generally slightly different, but that no *essential* difference exists between the respective pigments is proved by the fact that the constituents of both may be obtained crystallised in the same form.

In enterochlorophyll there is probably a more intimate union between the constituents than in plant chlorophyll.

All readings are reduced to wave-lengths, and the most important spectra mapped in the accompanying charts. The appearance of enterochlorophyll under the microscope in different cases is also shown in the accompanying drawing, as well as the crystals referred to above.

III. "Note on a Previous Paper." By G. H. Darwin, F.R.S., Fellow of Trinity College and Plumian Professor in the University of Cambridge. Received March 19, 1885.

The paper entitled "On the Stresses caused in the Interior of the Earth by the Weight of Continents and Mountains" ("Phil. Trans.,"

Part I, 1882, p. 187) has been found to be erroneous in certain points. The errors, however, do not touch the physical conclusions there attained. As this note has importance only in connexion with the paper, I proceed in the form of an appendix, without explanation of the notation.

In the first place—

Throughout the paper the normal stresses P, Q, R require an additional term W_i. The only function of these stresses used in obtaining physical results is P$-$R, and it remains unchanged when this correction is made.

The error takes its origin in § 1. Thomson's solution (1) when reduced to the form applicable to the incompressible solid, is the solution of the equations $-\frac{dp}{dx}+v\nabla^2\alpha=\frac{dW}{dx}$, and two others. The solution required is that of $-\frac{dp}{dx}+v\nabla^2\alpha=0$, and two others. The W involved in my solution is not the potential of a true bodily force, but only an "effective potential" producing the same strains as those due to the weight of the continents and mountains, but causing a different hydrostatic pressure. When, therefore, p is determined from Thomson's solution, that p is really equal to $p+W_i$ of the problem of the continents. Hence equation (3) should be $p=-\left(1+\frac{i}{I}\right)W_i$, instead of $p=-\frac{i}{I}W_i$. The correction to (3) must be carried on through the rest of the paper, and obviously it merely adds W_i to the stresses P, Q, R, leaving P$-$R, P$-$Q, Q$-$R unchanged.

The error would have been avoided had I, as suggested on p. 190, worked directly from the equations of equilibrium of the elastic incompressible solid, instead of from Thomson's solution.

When the solid is compressible, this method of "effective potential" (see "The Tides of a Viscous Spheroid," "Phil. Trans.," Part I, 1879, pp. 7–9) for including all the effects of gravitation is not applicable without certain additional terms in α, β, γ. Hence in § 10 where the solid is treated as being compressible the expressions for the stresses are incomplete. It will be found, however, that this incompleteness does not extend to the case of the mountains and valleys on the mean level surface, and that portion of the section remains correct. It would not be difficult to make the requisite corrections to the earlier part of the section, but I do not think it worth while to do so.

In the second place—

On p. 191 the following passage occurs:—

"It may be seen from considerations of symmetry that if W_i be a zonal harmonic, two of the principal stress-axes lie in a meridional

plane, and the third is perpendicular thereto. Moreover the greatest and least stress-axes are those which lie in that plane."

And in a foot-note on p. 200—

"It is easy to see that if a viscous sphere be deformed into the shape of a zonal harmonic, the flow of the fluid must be meridional, and from this we may conclude that in the elastic sphere the plane of greatest and least principal stresses must be also meridional. This has already been assumed to be the case in the present paper."

As one of the examiners for the Smith's Prizes at Cambridge, I have had placed before me an essay by Mr. Charles Chree, of King's College, in which he considers, amongst other points, the difference of principal stresses in an elastic sphere strained under the influence of the forces due to a potential expressed by the second zonal harmonic. In this essay Mr. Chree has pointed out that the conclusion thus arrived at by general reasoning is erroneous. His analytical treatment of the problem is entirely different from mine, and I cannot, therefore, avail myself of his actual work in amending the error which he has pointed out and corrected.

It is clear that, in the limiting case of the zonal harmonic where the surface becomes a series of parallel mountains and valleys on a flat surface, the principal stress parallel to the mountains must be zero, and the above reasoning has led to a correct conclusion.

But in the case of the second zonal harmonic, with either excess or deficiency of matter at the pole, there is a tendency for the equatorial regions to be either squeezed out or crushed in. Now an outward squeeze necessitates that the greatest pressure shall be perpendicular to the meridian, and this is contrary to the general conclusion quoted above. My error lay in overlooking this outward or inward tendency in the equatorial matter.

The conclusion is therefore wholly right in the case of the mountains and valleys, and at least partially wrong in the case of spheroidal deformation of the globe.

The data for examining into this question rigorously are given in my paper, and the best way of treating the matter is to rewrite § 5 on—

The State of Stress due to Ellipticity of Figure or to Tide-generating Forces.

When the effective disturbing potential W_i is a solid harmonic of the second degree, the solution found will give the stresses caused by oblateness or prolateness of the spheroid. It will also serve for the case of a rotating spheroid with more or less oblateness than is appropriate for the equilibrium figure. When an elastic sphere is under the action of tide-generating forces, the disturbing potential

is a solid harmonic of the second degree, and therefore the present solution will apply to this case also.

If we extract the case $i=2$ from Tables I, II, III, and put $i=2$ in (26), and substitute colatitude θ for latitude l, we have after some simple reductions—

$$\left.\begin{aligned} 19(\mathrm{P}-W_2) &= 16a^2-(19+3\cos 2\theta)r^2 \\ 19(\mathrm{R}-W_2) &= -32a^2+(29+3\cos 2\theta)r^2 \\ 19(\mathrm{Q}-W_2) &= 16a^2-(13+9\cos 2\theta)r^2 \\ 19\mathrm{T} &= 3\sin 2\theta r^2 \end{aligned}\right\} \quad . \quad . \quad . \quad (a).$$

[Note the introduction of W_2 in the P, Q, R, in accordance with the first correction.]

Let N_1, N_2, N_3, be the three principal stresses, each diminished by W_2, so that—

$$\left.\begin{aligned} \left.\begin{matrix}\mathrm{N}_1+W_2 \\ \mathrm{N}_3+W_2\end{matrix}\right\} &= \tfrac{1}{2}(\mathrm{P}+\mathrm{R})\pm\tfrac{1}{2}\sqrt{\{(\mathrm{P}-\mathrm{R})^2+4\mathrm{T}^2\}} \\ \mathrm{N}_2+W_2 &= \mathrm{Q} \end{aligned}\right\} \quad . \quad . \quad . \quad (b).$$

Then—

$$\left.\begin{aligned} \left.\begin{matrix}19\mathrm{N}_1 \\ 19\mathrm{N}_3\end{matrix}\right\} &= -8a^2+5r^2\pm 3\sqrt{\{64(a^2-r^2)^2+r^4-16r^2(a^2-r^2)\cos 2\theta\}} \\ 19\mathrm{N}_2 &= 16a^2-13r^2-9r^2\cos 2\theta \end{aligned}\right\} \quad (c).$$

Now let us find the surfaces, if any, over which $\mathrm{N}_2=\mathrm{N}_1$ or N_3. They are obviously given by—

$$24a^2-18r^2-9r^2\cos 2\theta=\pm 3\sqrt{\{64(a^2-r^2)^2+ \ . \quad . \quad \&c.\}}.$$

This easily reduces to—

$$r^2(1-\cos 2\theta)[32a^2-20r^2-9r^2(1+\cos 2\theta)]=0 \quad . \quad . \quad (d).$$

Thus the solutions are—

$$\left.\begin{aligned} r&=0 \\ \theta&=0 \text{ and } \pi \\ \text{and } 32a^2-20(x^2+y^2)-38z^2&=0 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (e).$$

By trial it is easy to see that at the centre and all along the polar axis $\mathrm{N}_2=\mathrm{N}_1$, and that inside of the ellipsoid $10(x^2+y^2)+19z^2=16a^2$, N_2 is greater than N_1, and outside it is less.

Hence inside of the ellipsoid $\mathrm{N}_2-\mathrm{N}_3$ and outside of it $\mathrm{N}_1-\mathrm{N}_3$ is the stress-difference. $\mathrm{N}_2-\mathrm{N}_3$ nowhere vanishes so long as N_2 is not equal to N_1, and $\mathrm{N}_1-\mathrm{N}_3$ vanishes where $r=\frac{2}{3}\sqrt{2}\,.\,a=\cdot 9428a$ and $\theta=0$, which is inside of the region for which $\mathrm{N}_1-\mathrm{N}_3$ is the stress-difference. This is the only point in the whole sphere for which the stress-difference vanishes.

The ellipsoid of separation cuts the sphere in colatitude 35° 16′.

If we put Δ for stress-difference, then between the centre and the ellipsoid—

$$19\Delta = 24a^2 - 18r^2 - 9r^2\cos 2\theta + 3\sqrt{\{64(a^2-r^2)^2 + r^4 - 16(a^2-r^2)r^2\cos 2\theta\}} \quad . \quad . \quad . \quad (f),$$

and between the polar surface regions and the ellipsoid—

$$19\Delta = 6\sqrt{\{64(a^2-r^2)^2 + r^4 - 16(a^2-r^2)r^2\cos 2\theta\}} \quad . \quad . \quad (g).$$

This last also holds for the whole polar axis, along which—

$$19\Delta = 6(8a^2 - 9r^2) \text{ or } 6(9r^2 - 8a^2).$$

[In the paper the form (g) for Δ was taken as applicable to the whole sphere; the maximum value of Δ arises from the form (g), and was therefore correctly computed.]

In order to find the actual value of Δ in any special case, we have to multiply the expression for Δ by appropriate factors, determined in the paper. For the present it will be convenient to omit these factors.

We may now from (f) and (g) determine the distribution of stress-difference throughout the sphere.

By computation and graphical interpolation I have drawn the annexed figure, showing the curves of equal stress-difference through-

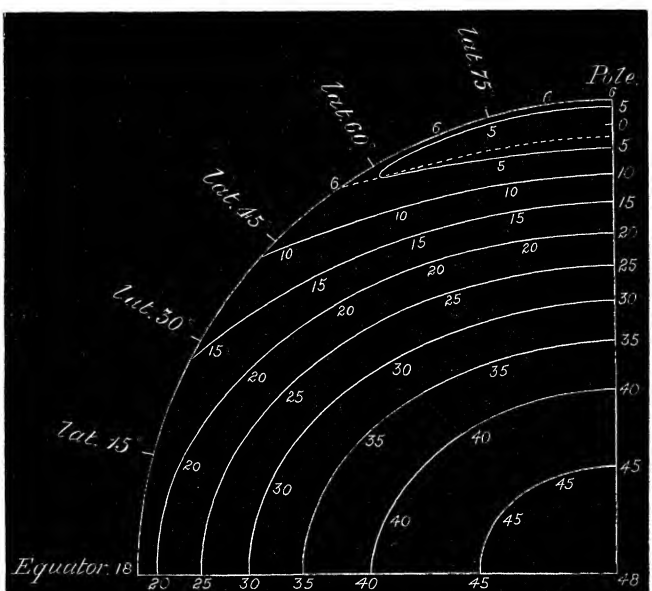

Diagram showing curves of equal Stress-difference due to the weight of 2nd harmonic inequalities or to tide-generating force.

out a meridional section of the sphere. The numbers written on the curves give the values of 19Δ, when the radius of the sphere is unity. The point marked 0 is that where Δ vanishes.

The dotted curve is the ellipse of separation (e) cutting the circle in colatitude 35° 16′.

Over the polar cap and at the surface 19Δ is constant and equal to 6; at the surface from colat. 35° 16′ to the equator 19Δ increases from 6 to 18, varying as the square of the sine of the colatitude.

At the centre 19Δ is 48, being eight times the polar superficial value.

Beginning with the first sentence of p. 203 the remainder of § 5 will hold good. It is well to observe, however, that where surface stress-difference is spoken of, it must be taken as referring to the polar caps only, the stress-difference at the equator being three times as great. It is worth while comparing the figure 1 of the paper (Plate 19) with the figure now given.

We now come to the case of—

The Stresses due to the even Zonal Harmonics.

The complete determination of the regions within which $N_2 - N_3$ and $N_1 - N_3$ are the proper measures of stress-difference might be somewhat difficult. As, however, these harmonics are only used for the determination of stress-difference in the equatorial regions, it is sufficient to find the boundary of the regions for that part of the sphere.

We see from (22) that $\sqrt{\{(P-R)^2+4T^2\}}$ only differs from $P-R$ by terms which depend on the square of the sine of the latitude.

Hence as far as the first power of $\sin l$ we have

$$N_1 = P - W_i, \quad N_2 = Q - W_i, \quad N_3 = R - W_i.$$

Therefore if we neglect terms depending on the square of the sine of the latitude, we have from (22),

$$\frac{IN_1}{r^{i-2}} = A_0 r^2 + B_0 a^2, \quad \frac{IN_2}{r^{i-2}} = G_0 r^2 + H_0 a^2, \quad \frac{IN_3}{r^{i-2}} = C_0 r^2 + D_0 a^2.$$

Then substituting, for A_0, B_0, &c., their values from (23), (24), (26), and effecting some easy reductions, we find,

$$\frac{IN_1}{r^{i-2}} = i^2(i+2)(a^2-r^2).$$

$$\frac{IN_2}{r^{i-2}} = i^2(a^2-r^2) + \frac{3i^2}{i-1}a^2.$$

$$\frac{IN_3}{r^{i-2}} = -[i(i+1)(i+2)+i](a^2-r^2) - \frac{i(i^2+3)}{i-1}a^2.$$

From this we see that N_1 is always positive but vanishes at the surface, N_2 is always positive but does not vanish at the surface, and N_3 is always negative.

Hence at the surface and for some distance beneath it, the stress-difference is $N_2 - N_3$; but below the surface at which N_1 becomes equal to N_2, we have $N_1 - N_3$ as the stress-difference.

This surface is determined by

$$i^2(i+2)(a^2-r^2)=i^2(a^2-r^2)+\frac{3i^2}{i-1}a^2.$$

whence
$$\frac{r^2}{a^2}=\frac{i^2-4}{i^2-1}$$

Solving for the successive even values of i, we find, when

$i=2$, $\frac{r}{a}=0$, as we already know.

$i=4$, $\frac{r}{a}=0{\cdot}8944$,

$i=6$, $\frac{r}{a}=0{\cdot}9562$,

$i=8$, $\frac{r}{a}=0{\cdot}9759$,

$i=10$, $\frac{r}{a}=0{\cdot}9847$.

In the paper $N_1 - N_3$ was always taken as being the stress-difference, and we now see that even when $i=4$, the region is very thin in which this is untrue and where $N_2 - N_3$ is the proper measure. For the higher harmonics it soon becomes negligeable.

This explains the transition from the incorrectness of the treatment in the paper of the case of the second harmonic to the correctness of the treatment of the mountain ranges.

On looking at § 7 and the accompanying figures we see that the maximum stress-difference occurs far within the region within which N_2 becomes the mean principal stress. Thus § 7 may be permitted to stand, save that in fig. 4, Plate 19, the ordinates of the curves $i=4$, $i=6$, &c., are to be slightly augmented at the surface where $r=a$. It is easy to see what small alterations are to be made in Table VI, and in the subsequent discussion, but clearly nothing material from a physical point of view need be amended.

It may be remarked in conclusion that, whilst it is proper to correct the mathematical errors in this paper, the physical conclusions remain untouched.

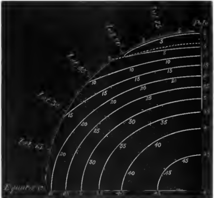

Diagram showing curves of equal Stress-difference due to the weight of 2nd harmonic inequalities or to tide-generating force.